probability that Bhim loses is 0.2

1.

Find the probability that, in 9 games, Bhim loses exactly 3 of the games, (a) (3) (b) fewer than half of the games. (2) Bhim attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05 Bhim and Joe agree to play a further 60 games. Calculate the mean and variance for the number of these 60 games that Bhim loses. (c) (2) Using a suitable approximation calculate the probability that Bhim loses more than 4 (d) games. (3) (Total 10 marks) 2. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company's website at their first attempt. Explain why the Poisson distribution may be a suitable model in this case. (a) (1) Find the probability that, in a randomly chosen 2 hour period, (b) (i) all users connect at their first attempt, (ii) at least 4 users fail to connect at their first attempt.

Bhim and Joe play each other at badminton and for each game, independently of all others, the

(5)

The company suffered from a virus infecting its computer system. During this infection it was found that the number of users failing to connect at their first attempt, over a 12 hour period, was 60.

(c) Using a suitable approximation, test whether or not the mean number of users per hour who failed to connect at their first attempt had increased. Use a 5% level of significance and state your hypotheses clearly.

(9) (Total 15 marks)

(3)

(3)

(2)

- **3.** A robot is programmed to build cars on a production line. The robot breaks down at random at a rate of once every 20 hours.
 - (a) Find the probability that it will work continuously for 5 hours without a breakdown.

Find the probability that, in an 8 hour period,

- (b) the robot will break down at least once,
- (c) there are exactly 2 breakdowns.

In a particular 8 hour period, the robot broke down twice.

(d) Write down the probability that the robot will break down in the following 8 hour period. Give a reason for your answer.

(2) (Total 10 marks)

4. A café serves breakfast every morning. Customers arrive for breakfast at random at a rate of 1 every 6 minutes.

Find the probability that

(a) fewer than 9 customers arrive for breakfast on a Monday morning between 10 am and 11 am.

(3)

The café serves breakfast every day between 8 am and 12 noon.

(b) Using a suitable approximation, estimate the probability that more than 50 customers arrive for breakfast next Tuesday.

(6) (Total 9 marks)

- 5. An administrator makes errors in her typing randomly at a rate of 3 errors every 1000 words.
 - (a) In a document of 2000 words find the probability that the administrator makes 4 or more errors.

The administrator is given an 8000 word report to type and she is told that the report will only be accepted if there are 20 or fewer errors.

(b) Use a suitable approximation to calculate the probability that the report is accepted.

(7) (Total 10 marks)

(3)

- 6. A cloth manufacturer knows that faults occur randomly in the production process at a rate of 2 every 15 metres.
 - (a) Find the probability of exactly 4 faults in a 15 metre length of cloth.
- (2)

(3)

(b) Find the probability of more than 10 faults in 60 metres of cloth.

A retailer buys a large amount of this cloth and sells it in pieces of length x metres. He chooses x so that the probability of no faults in a piece is 0.80

(c) Write down an equation for x and show that x = 1.7 to 2 significant figures.

(4)

The retailer sells 1200 of these pieces of cloth. He makes a profit of 60p on each piece of cloth that does not contain a fault but a loss of \pounds 1.50 on any pieces that do contain faults.

(d) Find the retailer's expected profit.

(4) (Total 13 marks)

(3)

(2)

7. A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

- (a) more than 2 daisies,
- (b) either 5 or 6 daisies.

The botanist decides to count the number of daisies, x, in each of 80 randomly selected squares within the field. The results are summarised below

$$\sum x = 295 \qquad \qquad \sum x^2 = 1386$$

- (c) Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places.
- (d) Explain how the answers from part (c) support the choice of a Poisson distribution as a model.

(1)

(3)

(e) Using your mean from part (c), estimate the probability that exactly 4 daisies will be found in a randomly selected square.

(2) (Total 11 marks)

8.	Each 0.000	cell of a certain animal contains 11000 genes. It is known that each gene has a probability 05 of being damaged.	
	A cel	l is chosen at random.	
	(a)	Suggest a suitable model for the distribution of the number of damaged genes in the cell.	(2)
	(b)	Find the mean and variance of the number of damaged genes in the cell.	(2)
	(c)	Using a suitable approximation, find the probability that there are at most 2 damaged genes in the cell. (Total 8 ma	(4) rks)
9.	A cal	I centre agent handles telephone calls at a rate of 18 per hour.	
	(a)	number of calls per hour handled by the agent.	(2)
	(b)	Find the probability that in any randomly selected 15 minute interval the agent handles(i) exactly 5 calls,	
		(ii) more than 8 calls.	(5)
	The a rando	agent received some training to increase the number of calls handled per hour. During a omly selected 30 minute interval after the training the agent handles 14 calls.	
	(c)	Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that the rate at which the agent handles calls has increased. State your hypotheses clearly.	
		(Total 13 ma	(6) rks)

10. (a) State two conditions under which a Poisson distribution is a suitable model to use in statistical work.

(2)

(5)

The number of cars passing an observation point in a 10 minute interval is modelled by a Poisson distribution with mean 1.

- (b) Find the probability that in a randomly chosen 60 minute period there will be
 - (i) exactly 4 cars passing the observation point,
 - (ii) at least 5 cars passing the observation point.

The number of other vehicles, other than cars, passing the observation point in a 60 minute interval is modelled by a Poisson distribution with mean 12.

(c) Find the probability that exactly 1 vehicle, of any type, passes the observation point in a 10 minute period.

(4) (Total 11 marks)

- **11.** An engineering company manufactures an electronic component. At the end of the manufacturing process, each component is checked to see if it is faulty. Faulty components are detected at a rate of 1.5 per hour.
 - (a) Suggest a suitable model for the number of faulty components detected per hour.

(1)

- (b) Describe, in the context of this question, two assumptions you have made in part (a) for this model to be suitable.(2)
- (c) Find the probability of 2 faulty components being detected in a 1 hour period.

(2)

	(d)	Find the probability of at least one faulty component being detected in a 3 hour period. (Total 8 mark	(3) (s)
12.	(a)	Write down the conditions under which the Poisson distribution may be used as an approximation to the Binomial distribution.	(2)
	A cal with	l centre routes incoming telephone calls to agents who have specialist knowledge to deal the call. The probability of the caller being connected to the wrong agent is 0.01	
	(b)	Find the probability that 2 consecutive calls will be connected to the wrong agent.	(2)
	(c)	Find the probability that more than 1 call in 5 consecutive calls are connected to the wrong agent.	(3)
	The c	call centre receives 1000 calls each day.	
	(d)	Find the mean and variance of the number of wrongly connected calls.	(3)
	(e)	Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent. (Total 12 mark)	(2) (s)

- **13.** The random variable *J* has a Poisson distribution with mean 4.
 - (a) Find $P(J \ge 10)$.

(2)

The random variable *K* has a binomial distribution with parameters n = 25, p = 0.27.

(b) Find $P(K \le 1)$.

- (3) (Total 5 marks)
- 14. (a) State the condition under which the normal distribution may be used as an approximation to the Poisson distribution. (1) Explain why a continuity correction must be incorporated when using the normal (b) distribution as an approximation to the Poisson distribution. (1) A company has yachts that can only be hired for a week at a time. All hiring starts on a Saturday. During the winter the mean number of yachts hired per week is 5. Calculate the probability that fewer than 3 yachts are hired on a particular Saturday in (c) winter. (2) During the summer the mean number of yachts hired per week increases to 25. The company has only 30 yachts for hire. (d) Using a suitable approximation find the probability that the demand for yachts cannot be met on a particular Saturday in the summer. (6) In the summer there are 16 Saturdays on which a yacht can be hired.
 - (e) Estimate the number of Saturdays in the summer that the company will not be able to meet the demand for yachts.

(2) (Total 12 marks)

- **15.** An estate agent sells properties at a mean rate of 7 per week.
 - (a) Suggest a suitable model to represent the number of properties sold in a randomly chosen week. Give two reasons to support your model.
 (3)
 (b) Find the probability that in any randomly chosen week the estate agent sells exactly 5 properties.
 (2)
 (c) Using a suitable approximation find the probability that during a 24 week period the estate agent sells more than 181 properties.
 (6) (Total 11 marks)

16. Breakdowns occur on a particular machine at random at a mean rate of 1.25 per week.

(a) Find the probability that fewer than 3 breakdowns occurred in a randomly chosen week.

(4)

Over a 4 week period the machine was monitored. During this time there were 11 breakdowns.

(b) Test, at the 5% level of significance, whether or not there is evidence that the rate of breakdowns has changed over this period. State your hypotheses clearly.

(7) (Total 11 marks)

17.	A manufacturer produces large quantities of coloured mugs. It is known from previous records that 6% of the production will be green.			
	A ra	ndom sample of 10 mugs was taken from the production line.		
	(a)	Define a suitable distribution to model the number of green mugs in this sample.	(1)	
	(b)	Find the probability that there were exactly 3 green mugs in the sample.	(3)	
	A ra	ndom sample of 125 mugs was taken.		
	(c)	Find the probability that there were between 10 and 13 (inclusive) green mugs in this sample, using		
		(i) a Poisson approximation,	(3)	
		(ii) a Normal approximation. (Total 13 ma	(6) (rks)	
18.	Acci	dents on a particular stretch of motorway occur at an average rate of 1.5 per week.		
	(a)	Write down a suitable model to represent the number of accidents per week on this stretch of motorway.	(-	
	Find	the probability that	(1)	
	(b)	there will be 2 accidents in the same week,	(2)	
	(c)	there is at least one accident per week for 3 consecutive weeks,	(3)	
	(d)	there are more than 4 accidents in a 2 week period.	(3)	

(3) (Total 9 marks) **19.** The random variable $X \sim B(150, 0.02)$.

	Use	a suitable approximation to estimate $P(X > 7)$. (7)	Cotal 4 marks)	
20.	The (a)	random variable X is the number of misprints per page in the first draft of a novel. State two conditions under which a Poisson distribution is a suitable model for X .	(2)	
	The that	number of misprints per page has a Poisson distribution with mean 2.5. Find the prob	ability	
	(b)	a randomly chosen page has no misprints,	(2)	
	(c)	the total number of misprints on 2 randomly chosen pages is more than 7.	(3)	
	The	first chapter contains 20 pages.		
	(d)	Using a suitable approximation find, to 2 decimal places, the probability that the ch will contain less than 40 misprints.	apter	
		(Te	(7) otal 14 marks)	

21. In a manufacturing process, 2% of the articles produced are defective. A batch of 200 articles is selected.

(a) Giving a justification for your choice, use a suitable approximation to estimate the probability that there are exactly 5 defective articles.

(5)

(b) Estimate the probability that there are less than 5 defective articles.

(2) (Total 7 marks)

22. The random variables *R*, *S* and *T* are distributed as follows

 $R \sim B(15, 0.3), \quad S \sim Po(7.5), \quad T \sim N(8, 2^2).$

Find

- (a) P(R = 5),
- (b) P(S = 5),
- (c) P(T=5). (1)

(Total 4 marks)

(2)

(1)

23. From company records, a manager knows that the probability that a defective article is produced by a particular production line is 0.032.

A random sample of 10 articles is selected from the production line.

(a) Find the probability that exactly 2 of them are defective.

(3)

On another occasion, a random sample of 100 articles is taken.

(b) Using a suitable approximation, find the probability that fewer than 4 of them are defective.

(4)

	At a	later date, a random sample of 1000 is taken.	
	(c)	Using a suitable approximation, find the probability that more than 42 are defective. (Total 13 m	(6) arks)
24.	Over mon	r a long period of time, accidents happened on a stretch of road at random at a rate of 3 per th.	
	Find	the probability that	
	(a)	in a randomly chosen month, more than 4 accidents occurred,	(3)
	(b)	in a three-month period, more than 4 accidents occurred.	(2)
	At a chos	later date, a speed restriction was introduced on this stretch of road. During a randomly en month only one accident occurred.	
	(c)	Test, at the 5% level of significance, whether or not there is evidence to support the claim that this speed restriction reduced the mean number of road accidents occurring per month.	(4)
	The	speed restriction was kept on this road. Over a two-year period, 55 accidents occurred.	
	(d)	Test, at the 5% level of significance, whether or not there is now evidence that this speed restriction reduced the mean number of road accidents occurring per month. (Total 16 m	(7) arks)

25. (a) State two conditions under which a random variable can be modelled by a binomial distribution.

(2)

In the production of a certain electronic component it is found that 10% are defective.

The component is produced in batches of 20.

(b) Write down a suitable model for the distribution of defective components in a batch. (1)

Find the probability that a batch contains

- (c) no defective components, (2)
- (d) more than 6 defective components.
- (e) Find the mean and the variance of the defective components in a batch.

(2)

(2)

A supplier buys 100 components. The supplier will receive a refund if there are more than 15 defective components.

(f) Using a suitable approximation, find the probability that the supplier will receive a refund.(4)

26. (a) Explain what you understand by a critical region of a test statistic.

(2)

The number of breakdowns per day in a large fleet of hire cars has a Poisson distribution with mean $\frac{1}{7}$.

(b) Find the probability that on a particular day there are fewer than 2 breakdowns.

Find the probability that during a 14-day period there are at most 4 breakdowns.

(3)

(3)

(c)

The cars are maintained at a garage. The garage introduced a weekly check to try to decrease the number of cars that break down. In a randomly selected 28-day period after the checks are introduced, only 1 hire car broke down.

(d) Test, at the 5% level of significance, whether or not the mean number of breakdowns has decreased. State your hypotheses clearly.

		(7)
(Total	15	marks)

Mino	or defects occur in a particular make of carpet at a mean rate of 0.05 per m^2 .	
(a)	Suggest a suitable model for the distribution of the number of defects in this make of carpet. Give a reason for your answer.	(3)
A can this c	rpet fitter has a contract to fit this carpet in a small hotel. The hotel foyer requires 30 m^2 of arpet. Find the probability that the foyer carpet contains	
(b)	exactly 2 defects,	(3)
(c)	more than 5 defects.	(3)
The c	carpet fitter orders a total of 355 m^2 of the carpet for the whole hotel.	
(d)	Using a suitable approximation, find the probability that this total area of carpet contains 22 or more defects. (Total 14 ma	(6) rks)
The r	random variable R has the binomial distribution B(12, 0.35).	
	Mind (a) A can this c (b) (c) The c (d) The r	 Minor defects occur in a particular make of carpet at a mean rate of 0.05 per m². (a) Suggest a suitable model for the distribution of the number of defects in this make of carpet. Give a reason for your answer. A carpet fitter has a contract to fit this carpet in a small hotel. The hotel foyer requires 30 m² of this carpet. Find the probability that the foyer carpet contains (b) exactly 2 defects, (c) more than 5 defects. The carpet fitter orders a total of 355 m² of the carpet for the whole hotel. (d) Using a suitable approximation, find the probability that this total area of carpet contains 22 or more defects. The random variable <i>R</i> has the binomial distribution B(12, 0.35).

(a) Find $P(R \ge 4)$. (2)

The random variable S has the Poisson distribution with mean 2.71.

(b) Find $P(S \le 1)$. (3)

The random variable *T* has the normal distribution $N(25, 5^2)$.

Find P($T \le 18$). (c)

(2) (Total 7 marks)

(6) (Total 14 marks)

29.	(a)	Write down two conditions needed to be able to approximate the binomial distribution by the Poisson distribution.	(2)
	A res Assu	searcher has suggested that 1 in 150 people is likely to catch a particular virus. ming that a person catching the virus is independent of any other person catching it,	
	(b)	find the probability that in a random sample of 12 people, exactly 2 of them catch the virus.	
			(4)
	(c)	Estimate the probability that in a random sample of 1200 people fewer than 7 catch the virus.	
		(Total 10 ma	(4) rks)
30.	Vehio	cles pass a particular point on a road at a rate of 51 vehicles per hour.	

Give two reasons to support the use of the Poisson distribution as a suitable model for the (a) number of vehicles passing this point.

(2)

Find the probability that in any randomly selected 10 minute interval (b) exactly 6 cars pass this point, (3) at least 9 cars pass this point. (c) (2) After the introduction of a roundabout some distance away from this point it is suggested that the number of vehicles passing it has decreased. During a randomly selected 10 minute interval 4 vehicles pass the point. Test, at the 5% level of significance, whether or not there is evidence to support the (d) suggestion that the number of vehicles has decreased. State your hypotheses clearly. (6) (Total 13 marks) 31. Write down the condition needed to approximate a Poisson distribution by a Normal (a) distribution. (1) The random variable $Y \sim Po(30)$. Estimate P(Y > 28). (b) (6) (Total 7 marks) 32. A doctor expects to see, on average, 1 patient per week with a particular disease. Suggest a suitable model for the distribution of the number of times per week that the (a) doctor sees a patient with the disease. Give a reason for your answer. (3) Using your model, find the probability that the doctor sees more than 3 patients with the (b) disease in a 4 week period. (4)

The doctor decides to send information to his patients to try to reduce the number of patients he sees with the disease. In the first 6 weeks after the information is sent out, the doctor sees 2 patients with the disease.

(c) Test, at the 5% level of significance, whether or not there is reason to believe that sending the information has reduced the number of times the doctor sees patients with the disease. State your hypotheses clearly.

Medical research into the nature of the disease discovers that it can be passed from one patient to another.

(d) Explain whether or not this research supports your choice of model. Give a reason for your answer.

(2) (Total 15 marks)

(6)

(2)

(4)

- **33.** A botanist suggests that the number of a particular variety of weed growing in a meadow can be modelled by a Poisson distribution.
 - (a) Write down two conditions that must apply for this model to be applicable.

Assuming this model and a mean of 0.7 weeds per m², find

- (b) the probability that in a randomly chosen plot of size 4 m^2 there will be fewer than 3 of these weeds.
- (c) Using a suitable approximation, find the probability that in a plot of 100 m^2 there will be more than 66 of these weeds.

(6) (Total 12 marks)

34. The continuous random variable X represents the error, in mm, made when a machine cuts piping to a target length. The distribution of X is rectangular over the interval [-5.0, 5.0].

Find

(a)
$$P(X < -4.2),$$
 (1)

(b)
$$P(|X| < 1.5).$$
 (2)

A supervisor checks a random sample of 10 lengths of piping cut by the machine.

	(c)	Find the probability that more than half of them are within 1.5 cm of the target length.	(3)
	If X rand	< -4.2, the length of piping cannot be used. At the end of each day the supervisor checks a om sample of 60 lengths of piping.	
	(d)	Use a suitable approximation to estimate the probability that no more than 2 of these lengths of piping cannot be used.	
		(Total 11 n	(5) narks)
35.	On a a rate	a typical weekday morning customers arrive at a village post office independently and at e of 3 per 10 minute period.	
	Find	the probability that	
	(a)	at least 4 customers arrive in the next 10 minutes,	(2)
	(b)	no more than 7 customers arrive between 11.00 a.m. and 11.30 a.m.	(3)
	The of 5	period from 11.00 a.m. to 11.30 a.m. next Tuesday morning will be divided into 6 periods minutes each.	
	(c)	Find the probability that no customers arrive in at most one of these periods.	(6)
	The	post office is open for $3\frac{1}{2}$ hours on Wednesday mornings.	
	(d)	Using a suitable approximation, estimate the probability that more than 49 customers arrive at the post office next Wednesday morning.	(7)
		(Total 18 m	(7) narks)

3

(a) Let X be the random variable the number of games Bhim loses. $X \sim B(9, 0.2)$ B1 $P(X \le 3) - P(X \le 2) = 0.9144 - 0.7382$ or $(0.2)^3 (0.8)^6 \frac{9!}{3!6!}$ M1 = 0.1762 = 0.1762awrt 0.176 A1

<u>Note</u>

1.

B1 – writing or use of B(9, 0.2)

M1 for writing/ using $P(X \le 3) - P(X \le 2)$ or $(p)^3 (1-p)^6 \frac{9!}{3!6!}$

A1 awrt 0.176

Special case : Use of Po(1.8)

can get B1 M1 A0 – B1 if written B(9, 0.2), M1 for $\frac{e^{-1.8} 1.8^3}{3!}$

or awrt to 0.161

If B(9, 0.2) is not seen then the only mark available for using Poisson is M1. (b) can get M1 A0 – M1 for writing or using $P(X \le 4)$ or may be implied by awrt 0.964

(b) $P(X \le 4) = 0.9804$

awrt 0.98 M1 A1 2

<u>Note</u>

M1 for writing or using $P(X \le 4)$ **A1** awrt 0.98

Special case : Use of Po(1.8)

can get B1 M1 A0 – B1 if written B(9, 0.2), M1 for $\frac{e^{-1.8} 1.8^3}{3!}$

or awrt to 0.161

If B(9, 0.2) is not seen then the only mark available for using Poisson is M1. (b) can get M1 A0 – M1 for writing or using $P(X \le 4)$ or may be implied by awrt 0.964

(c) Mean = 3 variance = 2.85, $\frac{57}{20}$ B1 B1

<u>Note</u>

B1 3 **B1** 2.85, or exact equivalent 2

(d)	Po(3)	poisson	M1	
	$P(X > 4) = 1 - P(X \le 4)$		M1	
	= 1 - 0.8153			
	= 0.1847		A1	3

Note

2.

M1 for using Poisson **M1** for writing or using $1 - P(X \le 4)$ NB $P(X \le 4)$ is 0.7254 Po(3.5) and 0.8912 Po(2.5) A1 awrt 0.185

Use of Normal

Can get M0 M1 A0 – for M1 they must write $1 - P(X \le 4)$ or get awrt 0.187

[10]

2.	(a)	Connecting occurs at random/independently, singly or at a constant rate	B1	1
		Note		
		B1 Any one of randomly/independently/singly/constant rate. Must have context of connection/logging on/fail		
	(b)	Po (8)	B1	
		Note		
		B1 Writing or using Po(8) in (i) or (ii)		
		(i) $P(X=0) = 0.0003$	1 1 A 1	
		$\frac{\text{Note}}{\text{Note}}$		
		M1 for writing or finding $P(X=0)$ A1 awrt 0.0003		
		(ii) $P(X \ge 4) = 1 - P(X \le 3)$ = 1 - 0.0424	M1	
		= 0.9576	A1	5
		<u>Note</u>		

M1 for writing or finding $1 - P(X \le 3)$ A1 awrt 0.958

(c)
$$H_0: \lambda = 4 (48)$$
 $H_1: \lambda > 4 (48)$ B1
N(48, 48) M1 A1

Method 1 $P(X \ge 59.5) = P\left(Z \ge \frac{59.5 - 48}{\sqrt{48}}\right) \qquad \frac{x - 0.5 - 48}{\sqrt{48}} = 1.6449 \text{ M1 M1 A1}$ $= P(Z \ge 1.66)$ = 1 - 0.9515 $= 0.0485 \qquad x = 59.9 \qquad A1$ 0.0485 < 0.05

Reject H ₀ . Significant. 60 lies in the Critical region	M1	
The number of failed connections at the first attempt has increased.	A1 ft	9

<u>Note</u>

B1 both hypotheses correct. Must use λ or μ M1 identifying normal A1 using or seeing mean and variance of 48 These first two marks may be given if the following are seen in the standardisation formula : 48 and $\sqrt{48}$ or awrt 6.93 M1 for attempting a continuity correction (Method 1: 60 ± 0.5 / Method 2: $x \pm 0.5$) M1 for standardising using their mean and their standard deviation and using either Method 1 [59.5, 60 or 60.5. accept $\pm z$.] Method 2 [($x \pm 0.5$) and equal to $a \pm z$ value) A1 correct z value awrt ±1.66 or ± $\frac{59.5-48}{\sqrt{48}}$, or $\frac{x-0.5-48}{\sqrt{48}}$ =1.6449 A1 awrt 3 sig fig in range 0.0484 – 0.0485, awrt 59.9 M1 for "reject H₀" or "significant" maybe implied by "correct contextual comment" If one tail hypotheses given follow through "their prob" and 0.05, p < 0.5If two tail hypotheses given follow through "their prob" with 0.025, *p* < 0.5 If one tail hypotheses given follow through "their prob" and 0.95, p > 0.5If two tail hypotheses given follow through "their prob" with 0.975, *p* > 0.5 If no H₁ given they get M0 A1 ft correct contextual statement followed through from their prob and H₁. need the words number of failed connections/log ons has increased o.e.

Allow "there are more failed connections"

NB A correct contextual statement <u>alone</u> followed through from their prob and H_1 gets M1 A1

[15]

3

3

3. (a) $Y \sim Po(0.25)$ B1 $P(Y=0) = e^{-0.25}$ M1 = 0.7788 A1

<u>Note</u>

B1 for seeing or using Po(0.25)

M1 for finding P(Y = 0) either by e^{-a} , where *a* is positive (*a* needn't equal their λ) or using tables if their value of λ is in them

Beware common Binomial error using, p = 0.05 gives 0.7738 but scores B0 M0 A0

A1 awrt 0.779

(b) $X \sim Po(0.4)$ B1 P(Robot will break down) = 1 - P(X = 0) $= 1 - e^{-0.4}$ M1 = 1 - 0.067032= 0.3297 A1

<u>Note</u>

B1 for stating or a clear use of Po(0.4) in part (b) or (c)

M1 for writing or finding 1 - P(X=0)

A1 awrt 0.33

(c) P(X=2) $\frac{e^{-0.4}(0.4)^2}{2}$ M1 = 0.0536 A1 2

<u>Note</u>

M1 for finding P(X=2) e.g $\frac{e^{-\lambda}\lambda^2}{2!}$ with their value of λ in or if their λ is in the table for writing P(X \le 2) - P(X \le 1)

A1 awrt 0.0536

(d)	0.3297 or answer to part (b)	B1ft		
	as Poisson events are independent	B1 dep	2	
	Note			
	1 st B1 their answer to part(b) correct to 2 sf or awrt 0.33			
	2^{nd} B1 need the word independent. This is dependent on them gaining the first B1			
	SC			
	Use of Binomial.			
	Mark parts a and b as scheme. They could get (a) B0,M0,A0 (b) B0 M1 A0			
	In part c allow M1 for ${}^{n}C_{2}(p)^{2} (1-p)^{n-2}$ with "their n" and "their p". They could get (c) M1,A0			
	DO NOT GIVE for $p(x \le 2) - p(x \le 1)$			
	In (d) they can get the first B1 only. They could get (d) B1B0		[10)]

4.	(a)	$X \sim \text{Po}(10)$		B1	
		P(X < 9)	$=\mathbf{P}(X \leq 8)$	M1	
			= 0.3328	A1	3

<u>Note</u>

B1 for using Po(10)

M1 for attempting to find $P(X \le 8)$: useful values $P(X \le 9)$ is 0.4579(M0), using Po(6) gives 0.8472, (M1).

A1 awrt 0.333 but do not accept $\frac{1}{3}$

(b) $Y \sim Po(40)$

M1 A1

Y is approximately N(40,40)

$$P(Y > 50) = 1 - P(Y \le 50)$$
 M1

$$= 1 - P \left(Z < \frac{50.5 - 40}{\sqrt{40}} \right)$$
 M1

$$= 1 - P(Z < 1.660..)$$
 A1

$$= 1 - 0.9515$$

= 0.0485 A1 6

N.B. Calculator gives 0.048437.

Poisson gives 0.0526 (but scores nothing)

<u>Note</u>

1st M1 for identifying the normal approximation

 1^{st} A1 for [mean = 40] and [sd = $\sqrt{40}$ or var = 40]

NB These two marks are B1 M1 on ePEN

These first two marks may be given if the following are seen in the standardisation formula : 40 and $\sqrt{40}$ or awrt 6.32

 2^{nd} M1 for attempting a continuity correction (50 or 30 \pm 0.5 is acceptable)

 3^{rd} M1 for standardising using their mean and their standard deviation and using either 49.5, 50 or 50.5. (29.5, 30, 30.5) accept \pm

2nd A1 correct z value awrt ± 1.66 or this may be awarded if see $\pm \frac{50.5 - 40}{\sqrt{40}}$ or $\pm \frac{29.5 - 40}{\sqrt{40}}$

3rd A1 awrt 3 sig fig in range 0.0484 – 0.0485

5. X = the number of errors in 2000 (a) **B**1 words $X \sim Po(6)$ SO $P(X \ge 4) = 1 - P(X \le 3)$ M1 = 1 - 0.1512= 0.8488awrt 0.849 A1 3 <u>Note</u> B1 for seeing or using Po(6) M1 for $1 - P(X \le 3)$ or 1 - [P(X = 0)]+ P(X=1) + P(X=2) + P(X=3)] A1 awrt 0.849 SC If B(2000, 0.003) is used and leads to awrt 0.849 allow B0 M1 A1 If no distribution indicated awrt 0.8488 scores B1M1A1 but any other awrt 0.849 scores B0M1A1 Y = the number of errors in 8000 words. (b) $Y \sim Po(24)$ so use a Normal approx M1 $Y \approx N(24, \sqrt{24}^2)$ A1 Require P(Y ≤ 20) = P $\left(Z < \frac{20.5 - 24}{\sqrt{24}}\right)$ M1 M1

[9]

= P(Z < -0.714)		A1	
= 1 - 0.7611		M1	
= 0.2389	awrt (0.237~0.239)	A1	7

[N.B. Exact Po gives 0.242 and no \pm 0.5 gives 0.207]

<u>Note</u>

1st M1 for identifying the normal approximation

 1^{st} A1 for [mean = 24] **and** [sd = $\sqrt{24}$ or var = 24]

These first two marks may be given if the following are seen in the standardisation formula :

$$\frac{24}{\sqrt{24}} \quad \text{or awrt 4.90}$$

 2^{nd} M1 for attempting a continuity correction (20/ 28 \pm 0.5 is acceptable)

3rd M1 for standardising using their mean and their standard deviation.

2nd A1 correct z value awrt ± 0.71 or this may be awarded if see $\frac{20.5 - 24}{\sqrt{24}}$ or $\frac{27.5 - 24}{\sqrt{24}}$

 4^{th} M1 for 1 – a probability from tables (must have an answer of < 0.5)

 3^{rd} A1 answer awrt 3 sig fig in range 0.237 - 0.239

[10]

6. (a)
$$X \sim Po(2)$$
 $P(X = 4) = \frac{e^{-2} \times 2^4}{4!} = 0.0902$ awrt 0.09 M1

A1 2

3

Note

M1 for use of Po(2) may be implied A1 awrt 0.09

(b) $Y \sim Po(8)$ B1 $P(Y > 10) = 1 - P(Y \le 10) = 1 - 0.8159$ = 0.18411... awrt 0.184 M1A1

<u>Note</u>

B1 for Po(8) seen or used M1 for $1 - P(Y \le 10)$ oe A1 awrt 0.184 of length x

(c) F = no. of faults in a piece of cloth

 $F \sim \operatorname{Po}\left(x \times \frac{2}{15}\right)$

 $e^{-\frac{2x}{15}} = 0.80$ M1A1 $e^{-\frac{2}{15} \times 1.65} = 0.8025..., e^{-\frac{2}{15} \times 1.75} = 0.791...$ **M**1 These values are either side of 0.80 therefore x = 1.7 to 2 sf A1 4 Note 1st M1 for forming a suitable Poisson distribution of the form $e^{-\lambda} = 0.8$ 1stA1 for use of lambda as $\frac{2x}{15}$ (this may appear after taking logs) 2nd M1 for attempt to consider a range of values that will prove 1.7 is correct **OR** for use of logs to show $lambda = \dots$ 2nd A1 correct solution only. Either get 1.7 from using logs or stating values either side S.C for $e^{-\frac{2}{15} \times 1.7} = 0.797... \approx 0.80$ $\therefore x = 1.7$ to 2 sf allow 2nd M1A0 Expected number with no faults = $1200 \times 0.8 = 960$ (d) M1 Expected number with some faults = $1200 \times 0.2 = 240$ A1 So expected profit = $960 \times 0.60 - 240 \times 1.50$, M1 A1 4 =£216 Note 1st M1 for one of the following 1200 p or 1200 (1 - p) where p = 0.8 or 2/15. 1st A1 for both expected values being correct or two correct expressions. 2nd M1 for an attempt to find expected profit, must consider with and without faults 2nd A1 correct answer only.

[13]

7. The random variable X is the number of daisies in a square. Poisson(3) **B**1 (a) $1 - e^{-3} (1 + 3 + \frac{3^2}{2!})$ $1 - P(X \le 2) = 1 - 0.4232$ **M**1 = 0.5768A1 3 $e^{-3}\left(\frac{3^5}{5!}+\frac{3^6}{6!}\right)$ $P(X \le 6) - P(X \le 4) = 0.9665 - 0.8153$ (b) M1 = 0.15122 A1 (c) $\mu = 3.69$ **B**1 $Var(X) = \frac{1386}{80} - \left(\frac{295}{80}\right)^2$ M1 accept $s^2 = 3.77$ = 3.73/3.72/3.71 A1 3 For a Poisson model, Mean = Variance ; For these data $3.69 \approx 3.73$ (d) \Rightarrow Poisson model **B**1 1 (e) $\frac{e^{-3.6875} 3.6875^4}{4!} = 0.193$ allow their mean or var M1 Awrt 0.193 or 0.194 A1 ft 2 8. *X* ~ B(11000, 0.0005) M1 A1 2 (a) M1 for Binomial, A1 fully correct These cannot be awarded unless seen in part a (b) $E(X) = 11000 \times 0.0005 = 5.5$ **B**1 Var (X) = $11000 \times 0.0005 \times (1 - 0.0005)$ = 5.49725**B**1 2 B1 cao

B1 also allow 5.50, 5.497, 5.4973, do not allow 5.5

[11]

9.

[8]

(c)	X ~ Po (5.5) P($X \le 2$) = 0.0884	M1A1 dM1 A1	4
	M1 for Poisson A1 for using Po (5.5) M1 this is dependent on the previous M mark. It is for attempting to find $P(X \le 2)$ A1 awrt 0.0884 Correct answer with no working gets full marks		
	Special case If they use normal approximation they could get M0 A0 M1 A0 if they use 2.5 in their standardisation.		
	NB exact binomial is 0.0883		
(a)	Calls occur singlyany two of the 3Calls occur at a constant rateonly need callsCalls occur independently or randomly.once	B1 B1	2
	B1 B1 They must use calls at least once. Independently and randomly are the same reason.Award the first B1 if they only gain 1 mark.Special case if they don't put in the word calls but write two correct statements award B0B1		
(b)	(i) $X \sim Po(4.5)$ used or seen in (i) or (ii) $P(X = 5) = P(X \le 5) - P(X \le 4)$ = 0.7020 - 0.5321	M1 M1	
	= 0.7029 - 0.3321 = 0.1708	A1	3
	correct answers only score full marks M1 Po (4.5) may be implied by them using it in their calculations in (i) or (ii) M1 for P(X < 5) – P(X < 4) or $\frac{e^{-\lambda} \lambda^5}{5!}$ A1 only awrt 0.171		
	(ii) $P(X > 8) = 1 - P(X \le 8)$	M1	
	= 1 - 0.9397 = 0.0403	A1	2
	correct answers only score full marks M1 for $1 - P (X \le 8)$ A1 only awrt 0.0403		

(c)	$ H_0: \lambda = 9 \ (\lambda = 1) $ $ H_1: \lambda > 9 \ (\lambda > 1) $	18) 18)	may use λ o	r μ	B1		
	X ~ Po (9) may	y be implied			B1		
$P(X \ge 14) =$ = 1 - 0.9261	$1 - P(X \le 13)$	$[P(X \ge 14) = 1 - 0.9261 = 0.0739]$ $P(X \ge 15) = 1 - 0.9780 = 0.0220$	att P($X \ge 14$)	P(<i>X</i> ≥ 15)	M1		
= 0.0739		$\operatorname{CR} X \ge 15$	awrt 0.0739				
0.0739 < 0.0)5	14 ≤ 15			A1		
	Accept H_0 . or i context from the	t is not significant or a correct sta eir values	tement in		M1		
	There is insuffi hour handled by	cient evidence to say that the nur y the agent has increased.	nber of calls I	per	A1	6	
	B1 both . Must or 18 and match	be one tail test. They may use λ oh H0 and H1	or μ and eithe	r 9			
	M1 Po (9) may M1 attempt to 1 A1 correct prob	be implied by them using it in the find $P(X \ge 14)$ eg $1 - P(X < 13)$ of pability or CR	eir calculation or $1 - P(X < 1)$	ns. 4)			
	To get the next2 marks the null hypothesis must state or imply that $(\lambda) = 9$ or 18						
	M1 for a correct statement based on their probability or critical region or a correct contextualised statement that implies that.						
	A1. This depends on their M1 being awarded for accepting H0.Conclusion in context. Must have calls per hour has not increased.Or the rate of calls has not increased.Any statement that has the word calls in and implies the rate not increasinge.g. no evidence that the rate of calls handled has increasedSaying the number of calls has not increased gains A0 as it does not imply rateNB this is an A mark on EPEN						
	They may also	attempt to find $P(X < 14) = 0.920$	51 and compa	re with 0.95	5		[13]

10.	(a)	Events occur at a constant rate.	any two of the 3	
		Events occur independently or randomly.		
		Events occur singly.		2
		B1B1 Need the word events at least once.		
		Independently and randomly are the same	e reason.	
		Award the first B1 if they only gain 1 ma	ırk	
		Special case. If they have 2 of the 3 lines witho	out the word events	
		they get B0 B1		

B1

(b) Let *X* be the random variable the number of cars passing the observation point.

Po(6)

(i)
$$P(X \le 4) - P(X \le 3) = 0.2851 - 0.1512 \text{ or } \frac{e^{-6} 6^4}{4!}$$
 M1

- (ii) $1 P(X \le 4) = 1 0.2851 \text{ or } 1 e^{-6} \left(\frac{6^4}{4!} + \frac{6^3}{3!} + \frac{6^2}{2!} + \frac{6}{1!} + 1 \right)$ M1 = 0.7149 A1 5
- B1 Using Po(6) in (i) or (ii)

M1 Attempting to find
$$P(X \le 4) - P(X \le 3)$$
 or $\frac{e^{-\lambda} \lambda^4}{4!}$

- A1 awrt 0.134
- M1 Attempting to find $1 P(X \le 4)$
- A1 awrt 0.715
- (c) P (0 car and 1 others) + P (1 cars and 0 other) B1 $= e^{-1} \times 2e^{-2} + 1e^{-1} \times e^{-2}$ $= 0.3679 \times 0.2707 + 0.3674 \times 0.1353$ M1A1 = 0.0996 + 0.0498 = 0.149A1 4 <u>alternative</u>

 $\overline{P_o(1+2)} = P_o(3) B1$ P(X = 1) = 3e⁻³ M1 A1 = 0.149 A1

B1 Attempting to find both possibilities. May be implied by doing $e^{-\lambda_1} \times \lambda_2 e^{-\lambda_2} + e^{-\lambda_2} \times \lambda_1 e^{-\lambda_1}$ any values of λ_1 and λ_2

M1 finding one pair of form $e^{-\lambda_1} \times \lambda_2 e^{-\lambda_2}$ any values of λ_1 and λ_2

- A1 one pair correct
- A1 awrt 0.149

Alternative. B1 for Po(3) M1 for attempting to find P(X = 1) with Po(3) A1 3e⁻³ A1 awrt 0.149

[11]

S2 Discrete distributions – Poisson

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11.	(a)	<i>X</i> ~ Po (1.5) need P	'o and 1.5	B1	1	
	(b)	Faultycomponents occur at a constant rate.any twoFaultycomponents occur independently or randomly.only notFaultycomponents occur singly.	o of the 3 ed faulty once	B1 B1	2	
	(c)	$P(X=2) = P(X \le 2) - P(X \le 1)$ or $\frac{e^{-1.5}(1.5)^2}{2}$		M1		
		= 0.8088 - 0.5578 = 0.251 at	wrt 0.251	A1	2	
	(d)	$X \sim Po(4.5)$ 4.5 may b $P(X \ge 1) = 1 - P(X = 0)$ $= 1 - e^{-4.5}$	e implied	B1 M1		
		= 1 - 0.0111 = 0.9889 a	wrt 0.989	A1	3	[8]
12.	(a)	If $X \sim B(n, p)$ and <i>n</i> is large, $n > 50$ <i>p</i> is small, $p < 0.2$ then <i>X</i> can be approximated by Po(<i>np</i>)		B1 B1	2	
	(b)	$P(2 \text{ consecutive calls}) = 0.01^2$ $= 0.0001$		M1 A1	2	
	(c)	$X \sim B(5, 0.01) $ may b $P(X > 1) = 1 - P(X = 1) - P(X = 0)$ $= 1 - 5(0.01)(0.99)^{4} - (0.99)^{5}$	e implied	B1 M1		
		= 1 - 0.0480298 0.95099 = 0.00098 awr	t 0.00098	A1	3	
	(d)	$X \sim B(1000, 0.01)$ may be implied by correct mean and Mean = $np = 10$ Variance = $np(1-p) = 9.9$	l variance	B1 B1 B1	3	
	(e)	<i>X</i> ~ Po(10)				
		$P(X > 6) = 1 - P(X \le 6)$ = 1 - 0.1301	M1			
		= 0.8699 a	wrt 0.870	A1	2	[12]

S2 Discrete distributions – Poisson

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13.	(a)	$P(J \ge 10) = 1 - P(J \le 9)$ = 1 - 0.9919 = 0.0081	or = $1 - P(J < 10)$ implies method awrt 0.0081	M1 A1	2	
	(b)	$P(K \le 1) = P(K = 0) + P(K = 1)$ '25' missing	both, implied below even with	M1		
		$= (0.73)^{25} + 25(0.73)^{24}(0.27)$ $= 0.00392$	clear attempt at '25' required awrt 0.0039 implies M	M1 M1	3	[5]
14.	(a)	$\lambda > 10$ or large	μ ok	B1	1	
	(b)	The Poisson is discrete and the r	normal is continuous.	B1	1	
	(c)	Let <i>Y</i> represent the number of ya P ($Y < 3$) = P($Y \le 2$) = 0.1247	achts hired in winter $P(Y \le 2) \& Po(5)$ awrt 0.125	M1 A1	2	
	(d)	Let X represent the number of ya N(25,25) all correct, can be in P(X > 30) $\approx P\left(Z > \frac{30.5 - 25}{5}\right) \pm 25$ $\approx P(Z > 1.1)$ $\approx 1 - 0.8643$ ≈ 0.1357	achts hired in summer $X \sim Po(25)$. mplied by standardisation below = standardise with 25 & 5; ±0.5 c.c. 1.1 'one minus' awrt 0.136	B1 M1;M1 A1 M1 A1	6	
	(e)	no. of weeks = 0.1357 × 16 = 2.17 or 2 or 3	ANS (d) × 16 ans > 16 M0A0	M1 A1ft	2	[12]
15.	Let	<i>X</i> represent the number of properties	es sold in a week			
	(a)	$\therefore X \sim P_0(7)$ <i>must be in part a</i>		B1		
			lowly, singly, of a constant set.	D1 D1	2	

Sales occur independently / randomly, singly, at a constant rate B1 B1 3 context needed once

(b)
$$P(X = 5) = P(X \le 5) - P(X \le 4)$$
 or $\frac{7^5 e^{-7}}{5!}$ M1
= 0.3007 - 0.1730
= 0.1277 A1
awrt 0.128

2

(c)	P (X > 181	$ \approx P (Y \ge 181.5) $ where $Y \sim N (168, 168) $	N (168,168)	B1		
	$=P\left(z \ge \frac{18}{2}\right)$	$\frac{1.5-168}{\sqrt{168}}$				
		± 0.5		M1		
		stand with μ and σ		M1		
		Give A1 for 1.04 or correct expression		Al		
	$= P (z \ge 1.0)$	4)				
	=1 - 0.8508	3 attempt correct area		M1		
		1 - p where $p > 0.5$				
	= 0.1492	awrt 0.149		A1	6	
						[11]

16. Let *X* represent the number of breakdowns in a week. (a) $X \sim P_o (1.25)$ **B**1 Implied P(X < 3) = P(0) + P(1) + P(2)or $P(X \le 2)$ M1 $=e^{-1.25}\left(1+1.25+\frac{(1.25)^2}{2!}\right)$ A1 = 0.868467A1 4 awrt 0.868 or 0.8685

H₀: $\lambda = 1.25$; H₁: $\lambda \neq 1.25$ (or H_o : $\lambda = 5$; H₁ : $\lambda \neq 5$) λ or μ B1 B1 (b) Let Y represent the number of breakdowns in 4 weeks Under H₀, $Y \sim P_0(5)$ B1 may be implied $P(Y \ge 11) = 1 - P(Y \le 10)$ M1or $P(X \ge 11) = 0.0137$ One needed for M $P(X \ge 10) = 0.0318$ = 0.0137 $\operatorname{CR} X \ge 11$ A1 0.0137 < 0.025, 0.0274 < 0.05, 0.9863 > 0.975, 0.9726 > 0.95 or $11 \ge 11$ M1 any .allow % ft from H_1 Evidence that the rate of breakdowns has changed / decreased B1ft 7 Context From their p

[11]

17.	(a)	Binomial	B1	1	
		Let X represent the number of green mugs in a sample			
	(b)	X ~ B (10, 0.06) may be implied or seen in part a	B1		
		P (X = 3) = ${}^{10}C_3(0.06)^3(0.94)^7$ ${}^{10}C_3(p)^3(l-p)^7$	M1		
		0.016808 awrt 0.0168	A1	3	
	(c)	Let X represent number of green mugs in a sample of size 125			
		(i) $X \sim P_0(125 \times 0.06 = 7.5)$ may be implied	B1		
		$(10 \le X \le 13) = P(X \le 13) - P(X \le 9)$ 0.9784 - 0.7764	M1		
		= 0.2020 <i>awrt 0.202</i>	A1	3	
		(ii) $P(10 \le X \le 13) \approx P(9.5 \le Y \le 13.5)$ where $Y \cup N(7.5, 7.05)$			
		7.05 9 5 7 3 5	BI B1		
		$= P\left(\frac{9.5 - 7.5}{\sqrt{7.05}} \le z \le \frac{13.5 - 7.5}{\sqrt{7.05}}\right)$	DI		
		± 0.5	M1		
		stand. both values or both correct expressions.	M1		
		$= P(0.75 \le z \le 2.26)$ <i>awrt</i> 0.75 <i>and</i> 2.26	A1		
		= 0.2147 awrt 0.214 or 0.215	A1	6	
					[13]
18.	(a)	Let X be the random variable the no. of accidents per week			

$X \sim Po(1.5)$ need poisson and λ must be in part (a) B1 1

(b) $P(X=2) = \frac{e^{-1.5}1.5^2}{2}$ = 0.2510 $\frac{e^{\mu}\mu^2}{2}$ or $P(X \le 2) - P(X \le 1)$ M1 awrt 0.251 A1 2

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(c)
$$P(X \ge 1) = 1 - P(X = 0) = 1 - e^{-1.5}$$
 correct exp awrt 0.777 B1
= 0.7769
P(at least 1 accident per week for 3 weeks)
= 0.7769³ (p)³ M1
= 0.4689 awrt 0.469 A1 3
The 0.7769 may be implied
(d) $X \sim Po(3)$ may be implied B1
 $P(X > 4) = 1 - P(X \le 4)$ M1

awrt 0.1847 A1 3

[9]

[4]

19.	$X = Po (150 \times 0.02) = Po (3)$	В	1, B1(dep))
	$P(X > 7) = 1 - P(X \le 7)$		M1	
	= 0.0119	awrt 0.0119	A1	
	Use of normal anneximation may amondo DO DO M1			

Use of normal approximation max awards B0 B0 M1 A0 in the use 1 - p(x < 7.5)

= 0.1847

$$z = \frac{7.5 - 3}{\sqrt{2.94}} = 2.62$$
$$p(x > 7) = 1 - p(x < 7.5)$$
$$= 1 - 0.9953$$
$$= 0.0047$$

20. Misprints are random / independent, occur singly B1, B1 (a) 2 in space and at a constant rate Context, any 2

(b)
$$P(X = 0) = e^{-2.5}$$
 M1
 $Po (2.5)$
 $= 0.08208..... = 0.0821$ A1 2

$$= 0.08208..... = 0.0821$$

(c) $Y \sim Po(5)$ for 2 pages **B**1 Implied

$$P(Y > 7) = 1 - P(X \le 7)$$

$$Use of 1 - and correct inequality$$
M1

$$= 1 - 0.8666 = 0.1334$$
 A1 3

(d)	For 20 pages, $Y \sim P_0$ (50)	B1	
	Y ~ N(50, 50) approx	B1	
	$P(Y < 40) = P(Y \le 39.5)$	M1	
	cc ±0.5		
	$= \mathbf{P}\left(Z \le \frac{39.5 - 50}{\sqrt{50}}\right)$		
	standardise above	M1	
	all correct	A1	
	$= P (Z \leq -1.4849)$	A1	
	<i>awrt</i> – 1.48		
	= 1 - 0.93 = 0.07	A1 7	
	0.07		
			[14]

21.	(a)	X ~ B(200, 0.02)	B1	
		Implied		
		<u>n large, P small</u> so X ~ Po (np) = Po (4) <i>conditions,</i> $P_0(4)$	B1, B1	
		$P(X = 5) = \frac{e^{-4} 4^5}{5!}$ $P(X \le 5) - P(X \le 4)$	M1	

(b)
$$P(X < 5) = P(X \le 4)$$

 $P(X \le 4)$
 $= 0.6288$ A1 2

[7]

A1 5

22.	(a)	$P(R = 5) = P(R \le 5) - P(R \le 4) = 0.7216 - 0.5155$	M1	
		Can be implied		
		= 0.2061	A1	2
		Answer 0.2061		
		(OR: ${}^{15}C_{5}(0.3)^{5}(0.7)^{10} = 0.206130)$		

[4]

(b)
$$P(S = 5) = 0.2414 - 0.1321 = 0.1093$$

Accept 0.1093 (AWRT) or 0.1094 (AWRT)
(OR: $\frac{7.5^5 e^{-7.5}}{5!} = 0.10937459....)$

(c)
$$P(T = 5) = 0$$
 B1 1
cao

23. Let x represent the number of defective articles
:.
$$X \sim B$$
 (10, 0.032)
(a) $P(X = 2) = \frac{10}{2} \sum_{2}^{4} \sum_{1}^{45} (0.032)^{2} (1 - 0.032)^{p}$ MI A1
Use of ${}^{n}C_{P}{}^{p}q^{n-5}$
All correct
 $= \frac{0.0355234....}{AWRT 0.0355}$ A1 3
(b) Large n, small p \Rightarrow Poisson approximation B1
with $\lambda = 100 \times 0.032 = 3.2$
 $P(X < 4) = P(X \le 3) = P(0) + P(1) + P(2) + P(3)$ M1
 $P(X \le 3)$ stated or implied
 $= e^{-3.2} \{1 + 3.5 + \frac{(3.2)^{2}}{2} + \frac{(3.2)^{3}}{6}\}$ A1
All correct
NB Normal Approx $\Rightarrow {}^{0}/_{4}$
 $= \frac{0.602519....}{AWRT 0.603}$ A1 4

(c)	$np \& nq \text{ both} > 5 \Rightarrow \text{Normal approximation}$	M1	
	N Approx With $np = 32$ and $npq = 30.976$	A1	
	Both		
	$P(X > 42) \approx P(Y > 42.5)$ where $Y \sim N(32, 30.976)$ <i>Standard</i>	M1	
	$= P(Z > \frac{42.5 - 32}{\sqrt{30.976}})$	A1	
	their np, \sqrt{npq} All correct		
	= P(Z > 1.8865) AWRT 1.89	A1	
	= <u>0.0294</u>	A1 6	
	0.0294 - 0.0297		[13]

24.	Let 2	<i>X</i> represent number of accidents/month $\therefore X \sim P_0(3)$	B1	
	(a)	$P(X > 4) = 1 - P(X \le 4); = 1 - 0.8513 = 0.1847$	M1; A1	3
	(b)	Let <i>Y</i> represent number of accidents in 3 months $\therefore Y \sim P_0(3 \times 3 = 9)$	B1	
		Can be implied		
		$P(Y > 4) = 1 - 0.0550 = \underline{0.9450}$	B1	2
	(c)	$H_0: \lambda = 3; H_1: \lambda < 3$	B1	
		both		
		$\alpha = 0.05$		
		$P(X \le 1/\lambda = 3) = 0.1991; > 0.05$	B1 M1	
		detailed; allow B0B1M1 (0.025) A0		
		:. Insufficient evidence to support the claim that the mean number of accidents has been reduced.	A1ft	4
		(NB: CR: $X \le 0$; $X = 1$ not in CR; same conclusion \Rightarrow B1, M1, A1)		

(d)	$H_0: \lambda 24 \times 3 = 72; H_1: \lambda < 72$	B1		
	can be implied $\lambda = 72$			
	$\alpha = 0.05 \Longrightarrow \text{CR: } \delta < -1.6449$ $both H_0 \& H_1$ -1.6449	B1 B1		
	Using Normal approximation with $\mu = \sigma^2 = 72$ <i>Can be implied</i>	B1		
	$\delta = \frac{55.5 - 72}{\sqrt{72}} = -1.94454\dots$	M1 A1		
	Stand. with ± 0.5 , $\mu = \sigma$ AWRT-1.94/5			
	Since -1.944 is in the CR, H_0 is rejected. There is evidence that the restriction has reduced the number of accidents	A1ft	7	
	Context & clear evidence			
	Aliter (d)			
	p = 0.0262 < 0.05			
	<i>AWRT 0.026 equn to -1.6449</i>			[16]
(a)	Fixed no of trials/ independent trials/ success & failure/ Probab of success is constant any 2	B1B1	2	
(b)	X is rv 'no of defective components $X \sim Bin(20,0.1)$	B1	1	
(c)	P(X=0) = 0.1216 = 0, 0.1216	M1A1	2	
(d)	$P(X > 6) = 1 - P(X \le 6) = 1 - 0.9976 = 0.0024$	M1A1	2	

Strict inequality & 1- with 6s, 0.0024

25.

(e)	$E(X) = 20 \times 0.1 = 2$ Var(X) = 20 × 0.1 × 0.9 = 1.8	B1 B1	2
(f)	$X \sim Bin(100, 0.1)$	B1	
	Implied by approx used		
	$X \sim P(10)$	B1	
	$P(X > 15) = 1 - P(X \le 15) = 1 - 0.9513 = 0.0487$	M1A1	
	Strict inequality and 1- with 15, 0.0487		
	(OR $X \sim N(10,9), P(X > 15.5) = 1 - P(Z < 1.83)$		
	= 0.0336 (0.0334) with 15.5	B1M1A1)	
	(OR $X \sim N(10, 10)$, $P(X > 15.5) = 1 - P(Z < 1.74)$ =0.0409 (0.0410) with 15.5	B1M1A1)	4

[13]

26.	(a)	<u>A range of values of a test statistic such that if a value of the to</u> obtained from a particular sample lies in the critical region,	est statis	stic	
		then the null hypothesis is rejected (or equivalent).		B1B1	2
	(b)	P(X < 2) = P(X = 0) + P(X = 1)	both	M1	
		$= e^{-\frac{1}{7}} + \frac{e^{\frac{1}{7}}}{7}$	both	A1	
		= 0.990717599 = 0.9907 to 4 sf awrt 0.991		A1	3
		$X \sim P(14 \times \frac{1}{7}) = P(2)$		B1	
		$P(X \le 4) = 0.9473$ Correct inequality, 0.9473		M1A1	3
		$H_0: \lambda = 4, H_1: \lambda < 4$		B1B1	
		Accept μ & H_0 : $\lambda = \frac{l}{7}$, H_1 : $\lambda < \frac{l}{7}$			
		$X \sim P(4)$ Implied		B1	
		$P(X \le 1) = 0.0916 > 0.05,$ Inequality 0.0916		M1A1	
		So insufficient evidence to reject null hypothesis Number of breakdowns has not significantly decreased		A1 A1	7

[15]

27.	(a)	No of defects in carpet area <i>a</i> sq m is distributed Po(0.05 <i>a</i>) <i>Poisson, 0.05a</i>	B1B1		
		Defects occur at a constant rate, independent, singly, randomly Any l	B1	3	
	(b)	$X \sim P(30 \times 0.05) = P(1.5)$	B1		
		$P(X=2) = \frac{e^{-1.5} \times 1.5^2}{2} = 0.2510$	M1A1	3	
		Tables or calc 0.251(0)			
	(c)	$P(X > 5) = 1 - P(X \le 5) = 1 - 0.9955 = 0.0045$ Strict inequality, 1-0.9955, 0.0045	M1M1A1	3	
	(d)	$X \sim P(17.75)$ Implied	B1		
		<i>X</i> ~ N(17.75, 17.75)	B1		
		Normal, 17.75			
		$P(X \ge 22) = P\left(Z > \frac{21.5 - 17.75}{\sqrt{17.75}}\right)$	M1M1		
		Standardise, accept 22 or ± 0.5			
		= -P(Z > 0.89)	A1		
		awrt 0.89		<i>.</i>	
		= 0.1867	AI	6	[15]
28.	(a)	$P(R \ge 4) = 1 - P(R \le 3) = 0.6533$ Require 1 minus and correct inequality	M1A1	2	
	(b)	$P(S \le 1) = P(S = 0) + P(S = 1), e^{-2.71} + 2.71e^{-2.71}, = 0.2469$ awrt 0.247	M1,A1,A1	3	
	(c)	$P(T \le 18) = P(Z \le -1.4) = 0.0808$	M1,A1	2	
		4 dp, cc no marks			[7]
					L' J

29. (a) n large, p small B1,B1 2 Let *X* represent the number of people catching the virus, **B**1 (b) $X \sim B\left(12, \frac{1}{150}\right)$ Implied $P(X=2) = C_2^{12} \left(\frac{1}{150}\right)^2 \left(\frac{149}{150}\right)^{10}, = 0.027$ M1A1,A1 4 Use of Bin including C_2^{12} , 0.0027(4) only (c) $X \sim \operatorname{Po}(np) = \operatorname{Po}(8)$ B1,B1 Poisson, 8 $P(X < 7) = P(X \le 6) = 0.3134$ M1A1 4

$$X \leq 6$$
 for method, 0.3134

[10]

30.	(a)	Vehicles pass at random / one at a time / independently / at a constant rate Any 2&context	B1B1dep	2	
	(b)	X is the number of vehicles passing in a 10 minute interval,			
		$X \sim \operatorname{Po}\left(\frac{51}{60} \times 10\right) = \operatorname{Po}(8.5)$	B1		
		Implied Po(8.5)			
		$P(X=6) = \frac{8.5^{6} e^{-8.5}}{6!}, = 0.1066 \text{ (or } 0.2562 - 0.1496 = 0.1066)$	M1A1	3	
		Clear attempt using 6, 4dp			
	(c)	$P(X \ge 9) = 1 - P(X \le 8) = 0.4769$	M1A1	2	
		Require 1 minus and correct inequality			
	(d)	H ₀ : $\lambda = 8.5$, H ₁ : $\lambda < 8.5$	B1ft,B1ft		
		One tailed test only for alt hyp			
		$P(X \le 4 \mid \lambda = 8.5) = 0.0744, > 0.05$	M1A1		
		$X \leq 4$ for method, 0.0744			
		(Or P($X \le 3 \mid \lambda = 8.5$) = 0.0301, < 0.05 so CR $X \le 3$ correct CR	M1,A1)		
		Insufficient evidence to reject H_0 ,	'Accept' M1		
		so no evidence to suggest number of vehicles has decreased.	Context A1ft	6	_
					[13]

[7]

31.	(a)	λ is large or $\lambda > 10$	B1	1
	(b)	$Y \sim N(30, 30)$ may be implied	B1	
		$P(Y > 28) = 1 - P(Y \le 28.5)$		
		$= 1 - P\left(Z \le \frac{28.5 - 30}{\sqrt{30}}\right)$	M1 A1	
		$= 1 - P(Z \le -0.273)$	A1	
		completely correct		
		= <u>0.606 - 0.608</u>	A1	6
		must be 3 or 4 dp		

32.	(a)	Po(1) Each patient seen singly <i>or</i> patients with disease seen randomly <i>or</i> patients seen at constant rate	B1 B1		
		or each patient assumed independent of the next	B1	3	
	(b)	$X \sim Po(4)$ may be implied $P(X > 3) = 1 - P(X \le 3)$ = 1 - 0.4335 = 0.5665	B1 M1 A1 A1	4	
	(c)	$H_0: \lambda = 6$ $H_1: \lambda < 6$ $P(X \le 2) = 0.0620$ $\alpha = 0.05 \Rightarrow$ critical region $X \le 1$ $0.0620 > 0.05$ 2 not in critical regionThe number of patients with the disease seenby the doctor has not been reduced	B1 B1 M1 A1 M1 A1	6	
	(d)	This <u>does not support the model</u> as the disease will occur in outbreaks; the patients seen by the doctor are unlikely to be independent of each other/don't occur singly	B1; B1	2	[15]

1

2

33.	(a)	Weeds grow independently, singly, randomly and at a $(waada/m^2)$	constant rate	D1 D1	2
		(weeds/m)	any 2	BIBI	Z
	(b)	Let <i>X</i> represent the number of weeds/ m^2			
		$X \sim Po(0.7)$, so in 4 m ² , $\lambda = 4 \times 0.7 = 2.8$		B1	
		P(Y < 3) = P(Y = 0) + P(Y = 1) + P(Y = 2)		M1	
		$= e^{-2.8} \left(1 + 2.8 + \frac{2.8^2}{2} \right)$		A1	
		= 0.46945		A1	4
	(c)	Let X represent the number of weeds per 100 m ²			

(c)	Let X represent the number of weeds per 100 m^2			
	$X \sim Po(100 \times 0.7 = 70)$	B1		
	$P(X > 66) \approx P(Y > 66.5)$ where $Y \sim N(70, 70)$	M1 M1 A1		
	$\approx \mathbf{P}\left(Z > \frac{66.5 - 70}{\sqrt{70}}\right)$	M1		
	$\approx P(Z > -0.41833) = 0.6628$	A1	6	[40]
				[12]

34.



(c)
$$Y = \text{no. of lengths with } |X| < 1.5$$
 $\therefore Y \sim B(10, 0.3)$ M1
 $P(Y > 5) = 1 - P(Y \le 5)$ M1

$$= 1 - 0.9527 = 0.0473$$
 A1 3

R = no. of lengths of piping rejected

$$R \sim B(60, 0.08) \implies R \approx \sim Po(4.8) \quad 4.8 \text{ or } 60 \times (a)$$
 B1 ft

$$P(R \le 2) = e^{-4.8} \left[1 + 4.8 + \frac{(4.8)^2}{2!} \right]$$
 Po and ≤ 2 , formula M1, M1 A1 ft

(ft for their λ if full expression seen)

$$= 17.32 \times e^{-4.8} = 0.1425...$$
(accept awrt 0.143) A1 cao5
[11]

35. (a)
$$X = \text{no. of customers arriving in 10 minute period}$$

 $X \sim \text{Po(3)} \quad P(X \ge 4) = 1 - P(X \le 3) =, \quad 1 - 0.6472 = 0.3528$ M1 A1 2
(b) $Y = \text{no. of customers in 30 minute period } Y \sim \text{Po(9)}$ B1
 $P(Y \le 7) = 0.3239$ M1 A1 3
(c) $p = \text{probability of no customers in 5 minute period = e^{-1.5}$ B1
 $C = \text{number of 5 minute periods with no customers}$
 $C \sim B(6, p)$ M1
 $P(C \le 1), = (1 - p)^6 + 6(1 - p)^5 p$ M1, M1 A1
 $= 0.59866...$
(accept awrt 0.599) A16

(d)
$$W =$$
 no. of customers on Wednesday morning

 $3\frac{1}{2}$ hours = 210 minutes $\therefore W \sim Po(63)$ '63' B1

Normal approximation $W \approx \sim N(63, (\sqrt{63})^2)$ M1 A1

$$P(W > 49) \approx P(W \ge 49.5)$$
 $\pm \frac{1}{2}$ M1

$$= P\left(Z \ge \frac{49.5 - 63}{\sqrt{63}}\right)$$

standardising M1

 $= P(Z \ge -1.7008)$ A1

[18]

1. This question was well answered by the majority of candidates with many scoring full marks. There were, of course, candidates who failed to score full marks. This was usually the result of inaccurate details, rather than lack of knowledge. In particular, manipulation of inequalities requires concentration and attention to detail. In part (a) the most common error seen was using $P(X = 3) = P(X \le 4) - P(X \le 3)$.

Parts (b) and (c) were usually correct. The most common error was to find $P(X \le 3)$ rather than $P(X \le 4)$ in part (b). A minority of candidates used the Normal as their approximation in part (d). The simple rule "*n* is large, *p* is small: use Poisson" clearly applies in this case.

2. The majority of candidates were familiar with the technical terms in part (a), but failed to establish any context.

Part (b) was a useful source of marks for a large proportion of the candidates. The only problems were occasional errors in detail. In part (i) a few did not spot the change in time scale and used Po(4) rather than Po(8). Some were confused by the wording and calculated P(X = 8) rather than P(X = 0). The main source of error for (ii) was to find $1 - P(X \le 4)$ instead of $1 - P(X \le 3)$.

In part (c) the Normal distribution was a well-rehearsed routine for many candidates with many candidates concluding the question with a clear statement in context. The main errors were

- Some other letter (or none) in place of λ or μ
- Incorrect Normal distribution: e.g. N(60, 60)
- Omission of (or an incorrect) continuity correction
- Using 48 instead of 60
- Calculation errors

A minority of candidates who used the wrong distribution (usually Poisson) were still able to earn the final two marks in the many cases when clear working was shown. This question was generally well done with many candidates scoring full marks.

3. Although there were a minority of candidates who were unable to identify the correct distribution to use the majority of candidates achieved full marks to parts (a) (b) and (c). Part (d) seemed to cause substantial difficulty. In part (a) the majority of candidates identified that a Poisson (rather than the Binomial) distribution was appropriate but some calculated the parameter as 2.5 or 4 rather than 0.4. A few used Po(1) and calculated P(Y = 5).

In part (b) and part (c) the most common error was to use Po(2.5). The majority of candidates were able to work out P(X>1) and P(X=2) using the correct Poisson formula. Many thought that their answer to part (c) was the correct solution while others used or multiplied their answers to both (b) and (c). Whether stating a correct or incorrect solution only a minority used the statistical term "independence" as the reason for their answer.

S2 Discrete distributions – Poisson

- 4. This question was accessible to the majority of candidates, with many gaining full marks. Most recognised the need to use a Poisson distribution in part (a) and translated the time of one hour successfully to a mean of 10. Common errors included using a mean of 6 or misinterpreting P(X < 9) as $P(X \le 9)$ or using $1 P(X \le 8)$. In part (b), a high percentage of candidates gained full marks for using a Normal approximation with correct working. Marks lost in this part were mainly due to using a 49.5 instead of 50.5 or no continuity correction at all. A small number of candidates wrote the distribution as B(240, 1/6) and translated this to N(40, 100/3).
- 5. Part (a) was answered well with the majority of candidates gaining full marks.

Part (b) was also a good source of marks for a large majority of the candidates. Common errors included using 23.9...for variance and 19.5 instead of 20.5. A sizeable minority of candidates used 21.5 after applying the continuity correction. A few candidates had correct working up to the very end when they failed to find the correct probability by not subtracting the tables' probability from 1.

6. Parts (a) and (b) were completed successfully by most candidates. The most common errors seen were using the wrong Poisson parameter or identifying the incorrect probability in part (b).

Part (c) proved to be a good discriminator with only those with good mathematical skills able to attain all the marks for this part of the question. Few candidates used the method given on the mark scheme and chose to use natural logarithms instead. Whilst this is an accepted method this knowledge is not expected at S2 and full marks were gained by the most able candidates using the given method.

In part (d) a few candidates seemed confused by this with some using 1.7 or 2/15 as a probability rather than the 0.8 given in the question, and far too many seemed unable to use 60p and £1.50 correctly when calculating the profit.

7. This question proved to be a good start to the paper for a majority of the candidates. There were many responses seen which earned full marks.

The most common errors in parts (a) and (b) concerned the routine manipulation of inequalities. In part (a) $1 - P(X \le 1)$ was often seen and in (b), while most candidates agreed that $P(5 \le X \le 6)$ was the required probability, with many then choosing the standard technique of $P(X \le 6) - P(X \le 4)$, there were candidates who proceeded with a variety of methods. Incorrect expressions such as $P(X \le 6) - P(X \le 4)$ were seen not infrequently. A correct but inefficient method which was commonly used included:

 $P(X = 5) + P(X = 6) = (P(X \le 6) - P(X \le 5)) + (P(X \le 5) - P(X \le 4))$

Part (c) was poorly answered. There were a significant minority of candidates who obtained a 'correct' answer for the mean in part (c), but who nevertheless lost the mark because their answer was not written, as instructed, correct to 2 decimal places. Many candidates were unable to calculate the variance. There were a variety of incorrect formulae used.

The general response to (d) was good, although many candidates simply gave the response that is appropriate for a more frequent type of question on the Poisson distribution requiring comment: ("singly/independently/randomly/constant rate").

Part (e) was particularly well done. Even the minority who struggled, or even omitted, some of the earlier parts of the question were able to gain both marks in part (e).

8. This question was generally answered well. A few candidates put the Poisson for (a) and then used Variance = Mean to got 5.5 for the variance. Some candidates rounded incorrectly giving an answer of 5.49 for the variance.
 Part (c) was generally answered correctly although a minority of candidates used the normal

approximation – most used 2.5 in their standardisation and so got 1 mark out of the 4.

- 9. Part(a) was done well generally although a reference to 'calls' was not made by a few candidates. Weak candidates talked about the Poisson needing large numbers and others seemed to not understand what was required at all; writing 'quick and easy'. Part (b) was done correctly by the majority of candidates. A few did not use Po(4.5) in (i) and a number used P(X > 8) = 1 P($X \le 7$) in (ii).In part (c) weaker candidates did not use λ for their hypotheses nor did they use Po(9). Some hypotheses had $\lambda = 3.5$ and $\lambda \ge 3.5$. Two tail tests were often suggested. Most candidates got to 0.0739 and only a few candidates used the critical value route. Only the able candidates got the interpretation of the significance test correct. Weak candidates generally only considered whether it was significant or not, with mixed success. They rarely managed to interpret correctly in context.
- **10.** Most candidates were able to attempt part (b) successfully as these were fairly standard calculations. However, when required to apply Poisson probabilities to a problem it was only the better candidates who attained any marks in part (c).
 - (a) A sizeable proportion of candidates, whilst having learnt the conditions for a Poisson distribution, failed to realise that this applied to the <u>events</u> occurring. There were references to 'trials' and 'things' in some solutions offered.
 - (b) (i) Most candidates recognised Po(6) and were able to answer this successfully, either from the tables or by calculation. Common errors were using incorrect values from the table or calculating the exact value incorrectly.
 - (ii) Although many candidates attained full marks for this part, some were unable to express 'at least 5' correctly as an inequality and used $P(X \le 5)$.
 - (c) Many of the successful candidates used a Po(3) to give a correct solution. Of the rest most candidates failed to realise that there were two ways for exactly one vehicle to pass the point and so only performed one calculation. This was often for P(1 car) and P(1 other vehicle), which were then added together. However there were some candidates who gained the correct answer via this method.
- 11. This question was quite well done. Most candidates were aware of the conditions for a Poisson distribution but many lost marks as they did not answer the question in context, although the examination question actually specified this. A few did not realise that independent and random were the same condition. Cumulative probability tables were well used in part (c) and there were many accurate solutions using the Poisson formula.

Most candidates used a mean of 4.5 in part (d) and there were many accurate results.

12. This question was accessible to most candidates. Parts (a), (d) and (e) were generally well answered with a small proportion of candidates using Po(10) in part (d) and thus quoting that the mean and variance were the same. Part (b) caused a few problems as some used Po(0.1) and found P(X = 2) or used $X \sim B(2, 0.1)$. In part (c) some candidates still did not correctly use P(X > 1) = 1 - P(X < 1). A similar mistake occurred in part (e) where they used $P(X > 6) = 1 - P(X \le 6)$.

Candidates need to be reminded of the rubric on the front of the question paper. It does say 'appropriate degrees of accuracy'. Many rounded too early and did not realise that an answer to 1sf is not accurate enough. Answers of 0.001 were common in part (c)

- 13. This was usually completely correct with very few errors.
- 14. It was disappointing to find that a large number of candidates failed to attain both of the first 2 marks available. These were often the only marks lost by some, since the majority of candidates achieved most or all marks. In part (d) most candidates did attempt an approximation, although a minority calculated an exact binomial. Again, the common errors were to fail to use a continuity correction and the standard deviation when using the approximation and then not using the $1 \Phi(z)$. The simple calculation of 16 x the answer to part (d) was performed correctly by the majority of candidates attempting this part of the question. A common error was to attempt a binomial probability.
- **15.** The majority of candidates knew the conditions for the Poisson distribution but many did not get the marks because they failed to put them into context. As in many previous series, it was very common for candidates to repeat at least some of these conditions parrot-fashion preceded by "events occur" or "it occurs". Other common errors were listing randomness and independence as separate reasons and citing the fixed time period and lack of an upper limit as reasons. Quite a number failed to mention the parameter for the distribution. The majority of candidates answered part (b) correctly. Most candidates answered part (c) correctly. Where marks were lost it was usually through failing to use a continuity correction rather than applying it wrongly.
- 16. Most candidates answered Part (a) correctly. A small number of candidates calculated the probability for less than or equal to 3 although a minority thought that dividing by 0! in P(X = 0) gave zero. In part (b) carrying out the hypothesis test was more challenging though there was clear evidence that candidates had been prepared for this type of question. However, using *p* instead of λ or μ , when stating the hypotheses, was often seen and incorrectly stating H1 as $\lambda > 1.25$ or 5 also lost marks. Many candidates calculated $P(X \le 11)$ instead of looking at $P(X \ge 11)$. A diagram would have helped them or the use of the phrase "a result as or more extreme than that obtained". Those who used the critical region approach made more errors. Some candidates correctly calculated the probability and compared it with 0.025 but were then unsure of the implications for the hypotheses. A few candidates used a 2-tailed hypothesis but then used 0.05 rather than 0.025 in their comparison. Most candidates gave their conclusions in context.

S2 Discrete distributions – Poisson

- 17. This was well answered by almost all candidates and many correct solutions were seen. A few candidates tried to use Poisson rather than Binomial for parts (a) and (b). In part (b) a few candidates used B(10, 0.6) instead of B(10, 0.06). In part (c)(i) most errors occurred because candidates did not understand what was meant by "between 10 and 13 inclusive" The most common wrong answer was in using $P(10 \le X \le 13) = P(X \le 13) P(X \le 10)$ instead of $P(X \le 13) P(X \le 9)$ Another fairly common error was using $P(X \le 13) (1 P(X \le 10))$ Some candidates tried to use a continuity correction in this Poisson approximation. Part (c)(i) was often correct the most common errors being to use 7.5 instead of 7.05 for the variance and to use an incorrect continuity correction.
- **18.** Many candidates did well on this question and gained all 9 marks.

In part (a) nearly all candidates realised the Poisson distribution was appropriate but not all stated the parameter of 1.5. Often this was stated in part b which did not gain the marks.

In part (c) many candidates used a Poisson distribution with a parameter of 4.5 rather than cubing the probability of $X \ge 1$ from a Poisson 1.5.

Part (d) was done very well by all candidates; the main error being the statement $P(X > 4) = 1 - P(X \le 3)$

- 19. Most candidates correctly used a Po(3) distribution although a significant minority attempted to used a Normal distribution. The most common error was using $P(X > 7) = 1 P(X \le 6)$
- 20. Many candidates did not achieve any marks in part (a) as they failed to give conditions in context, events being the most common error seen. Part (b) was done well and if candidates lost a mark then it was usually the final mark due to accuracy. A common answer was 0.082. Part (c) was generally completed satisfactorily, but there were a number of candidates who struggled with the inequalities. A common error was $P(Y > 7) = 1 P(T \le 6)$. Diagrams were again in evidence; these candidates were perhaps the least likely to make mistakes with the inequalities. Some candidates calculated a probability using P(2.5) and then squared their answer. The overall response to part (d) was good. However, only a minority scored full marks. Most candidates failed to implement the instruction to write their answer "to 2 decimal places". There were other errors; omission of the continuity correction or the wrong version (40.5), confusion between variance and standard deviation, and problems dealing with a negative *z*-value.
- **21.** The overall response was good, with a large number of candidates scoring at least five out of the seven marks. However, a small number of candidates chose to ignore the instructions to "use a suitable approximation". Most candidates were familiar with the conditions required for the Poisson approximation to the Binomial in part (a). However, a small number of candidates quoted this correct reason, but used this as justification for a Normal approximation. Not all candidates using the Poisson distribution earned the second mark. A final answer of 0.156 was fairly common, particularly amongst those who had used the Poisson formula rather than the cumulative tables.

The response to part (b) was excellent. There were a large number of perfect answers. A small number of candidates ignored the instruction to approximate and continued the use the Binomial

distribution. Cumulative tables are not available for this particular distribution, so candidates calculated five separate probabilities using the Binomial formula and then added, resulting in many of them able to obtain the correct answer using this method.

- **22.** Candidates knew how to answer parts (a) and (b) but many did not work to sufficient accuracy. If they used their calculator instead of the tables they were expected to give their answer to the same accuracy as the tables. Too many of them did not read part(c) carefully enough. The random variable *T* was defined to be normally distributed and thus P(T=5) = 0.
- **23.** This question was a good source of marks for many of the candidates, with many of them gaining full marks. For those that did not gain full marks, the common errors were premature approximation; wrong interpretation of 'fewer than 4'; ignoring the continuity correction and in part (c) using a Poisson approximation and then a normal approximation to this Poisson approximation.
- 24. For those candidates that could interpret 'more than 4 accidents occurred' correctly parts (a) and (b) were a good source of marks. Part (b) was often well answered and many candidates gained full marks. In part (c) incorrect hypotheses and ignoring the continuity correction were the common errors coupled with poor use of the appropriate significance test. Candidates need to have a simple algorithm at their fingertips to deal with tests of significance.
- **25.** This was a good source of marks for a large majority of candidates. Many were able to write down two conditions for a Binomial distribution, with some writing down all four conditions for good measure. However some candidates muddled up the concepts of *trial, event* and *outcome*. The response of 'a fixed number of events' was given no credit. Parts (b), (c), (d) and (e) were usually well answered. In part (f), successful candidates either applied a Po (10) or a N(10, 9) approximation. Some of the candidates who used the N(10, 9) approximation did not apply the correct continuity correction.
- **26.** In part (a) many candidates struggled to explain the concept of a critical region, although some gave a correct definition as the range of values where the null hypothesis is rejected. Many correct solutions were seen for parts (b) and (c). However the weaker candidates were not able to translate the concept 'at most 4 breakdowns' to the correct inequality. In part (d), as with Q3, many candidates successfully performed the required hypothesis test using a probability method. Again, there was a sizeable number of candidates who incorrectly found P(X=1) and compared this probability with the significance level. Again, a minority of candidates decided to approach this question using the critical region strategy. Marks were lost if candidates did not give evidence of their chosen critical region.

- **27.** This was a well answered question and high marks were frequently being scored. In part (a), many candidates chose the correct Poisson model and gave a correct reason in the light of the problem posed. Many correct solutions were seen in parts (b) and (c). In part (d), many candidates found the correct Normal approximation to the Poisson distribution. The most common error was the incorrect application of the continuity correction with weaker candidates generally losing the final two accuracy marks. In part (a), a small minority of candidates incorrectly chose a Binomial model and applied this model throughout the question, thereby losing a considerable number of marks.
- **28.** The whole question was generally very well answered, by far the most common error was the use of a continuity correction in part (c). A small number of candidates didn't realise that part (b) required the sum of two probabilities.
- **29.** This was a well answered question with many candidates scoring full marks. In part (a), many candidates realised the conditions of a *large value of n* and a *small value of p* when approximating the Binomial Distribution by the Poisson distribution. One common error in part (b) was for candidates to apply the Poisson approximation when the number of trials was only twelve, even though these candidates were able to write down the appropriate conditions in part (a).
- **30.** Candidates were able to express two conditions for a Poisson distribution in context with vehicles passing by a particular point on the road. Many candidates then answered part (b) and (c) correctly. In part (d) a majority of candidates was able to give a full solution by either using a probability or critical region approach to their hypothesis test.
- **31.** This question was generally well answered with many candidates gaining full marks. In part (b) a minority of candidates standardized correctly but then found the incorrect area.
- **32.** This question was well answered with many candidates gaining 14 or 15 marks. In part (a) most candidates spotted the Poisson distribution but few related their reason to the context of the question. In part (d) the majority of candidates realized that the Poisson was no longer appropriate, but some failed to give a reason which related the breakdown of one of the Poisson conditions to the context of the question.
- **33.** Most candidates scored well on this question, but too many lost marks in part (a) by not giving their answer in context. Too many candidates lost a mark by not answering part (b) to the required level of accuracy. Candidates were obviously more at ease in part (c) where they were able to leave the answer as they found it from tables.

- **34.** No Report available for this question.
- **35.** No Report available for this question.